THE AIMS OF STUDYING PLANE GEOMETRY AND HOW TO ATTAIN THEM.*

BY E. P. SISSON.

"There is no royal road to geometry," says Euclid. Plato had written over the porch of his famous school, "Let no one who is unacquainted with geometry enter here." An old poet has written, "Without enthusiasm, no mathematics."

Geometry is a human book—not divine; therefore a very imperfect book. Geometry is the product of the human mind, and not of the hand; therefore the subject concerns the intellect, and is not mechanical.

The aims of studying geometry in our schools are many; but in my opinion the most important purpose is the training of the reasoning faculty, which gives the student the power of clear and accurate thinking, and the power to express his thought in concise and elegant language. A story is related by Stobæus which runs thus: A youth who had begun to read geometry with Euclid, when he had learned the first proposition, inquired, "What do I get by learning these things?" Euclid at once called his slave and said, "Give him three pence, since he must make gain out of what he learns."

Another purpose of studying plane geometry is to obtain the knowledge by which the student can measure lines, angles and surfaces. The student of plane geometry lays the foundation necessary for the study of solid geometry, and the study has also a practical application in the study of the sciences, especially chemistry and physics. The study of geometry gives to the mind what the strope does to the razor—a keen, cutting edge.

In my opinion, fully three fourths of the value of the study of geometry is disciplinary. I well know that this is not in harmony with the modern thought and method. We meet those teachers occasionally who say that the study of geometry is becoming stale and uninteresting, and therefore we must have something new, in method at least. I profoundly pity such teachers. Our experience and study leads us very emphatically

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to the conclusion that science precedes all art, and that theory goes before practice and the general formula includes all particulars. The mind is strengthened far more by the consideration of general propositions than by the consideration of isolated ones. The mind must ever direct the hand.

How shall we as teachers present the subject of plane geometry so our pupils will get the most out of the study that will be profitable to them in after years? Any method must be very flexible, in order that it can be adapted to each individual teacher. I venture to suggest a method something like the following. First, decide on the text-book to be used. Second, commit the definitions, absolutely, of the fundamental ideas in that text, such as point, line, extension, direction, geometrical figure, angle (both as a figure and as a magnitude). Be sure that the class understand clearly and accurately the language used and can illustrate each definition. Third, commit the corollaries from the definitions. Fourth, the axioms and postulates.

I wonder how many teachers ever asked their pupils to state in good English what they understand by mathematical reasoning. I have never read in any geometry a definition of mathematical reasoning. In my mind, the definition is something like this: Mathematical reasoning consists of a series of mental steps by which the mind passes from the hypothesis of a proposition to a conclusion, and each step is made up of two distinct parts; first, the statement of a fact that leads the mind toward the conclusion, and, second, the authority for such statement, which must always precede the proposition under consideration, and the authority must be a definition, corollary, axiom, postulate, or a proposition previously demonstrated. "Is evident" is never an authority for any step in a mathematical demonstration. Olney says, "The mathematical reasoning by which the truth or falsity of a proposition is made to appear is called a mathematical demonstration;" but, mark you, this does not define mathematical reasoning.

We find no two authors agreeing in the definitions, corollaries deduced from the definitions, axioms, postulates, and order of propositions; therefore the demonstrations will differ in the method, order, and number of steps required, in accordance with the text-book used. The following will illustrate how

authors differ in the first proposition. The first proposition in Wells is: If two straight lines intersect, the vertical angles are equal. In Beman and Smith: All right angles are equal. In Olney: At any point in a straight line one perpendicular can be erected to the line, and only one which shall lie on the same side of the line. Robbins: A right angle is equal to half a straight angle. Lyman: If two triangles have two sides and the included angle of one respectively equal, the triangles are congruent. Euclid: Upon a given segment line construct an equilateral triangle. Stewart: To make two straight lines coincide in direction. Gillette: All the radii of a circle are equal. Newcomb: A straight line can be bisected in only a single point. Pettee: If one straight line meets another, the adjacent angles are supplementary. This is sufficient to illustrate my point. We find almost as great a difference in the definitions, postulates, corollaries, axioms, as given by different authors.

After having decided upon the text-book to be used, have the class work from four to six weeks without any text-book except as they use it for the definitions, axioms, postulates, and corollaries. During this time, dictate from ten to fifteen fundamental propositions that can be easily demonstrated by the use as authorities of the definitions, corollaries, axioms, and postulates already given; and have the class go over each demonstration carefully and thoroughly in order to show to the student the "nature of a geometric proof, and to lead him by easy steps to appreciate the logic of geometry."

After this, follow the text-book closely, and have the class see that the author is consistent in his development of the subject in accordance with his definitions, axioms, postulates, and corollaries, and order of propositions as dictated and given by the author used.

The recitation should be the opportunity for the teacher to direct the work of the student, but, under no conditions, to *do* the work *for* the student. Too often the recitation period is spent in talking about the subject, which is time worse than wasted. In general, no student in geometry should be allowed the privilege of the class without having first studied hard, from one to two hours if necessary, upon the lesson previously assigned. Give the student just as little help as possible, and eliminate all material that is not absolutely essential in each

demonstration. This will not make the subject of geometry interesting or a pleasure to every student. Some will dislike it for a time, and a few will fail; but we believe in the end more will be helped than discouraged by this method, and a much higher grade of scholarship will be attained.

COLGATE ACADEMY,
HAMILTON, N. Y.

MODERN TENDENCIES IN THE TEACHING OF MATHEMATICS.*

BY ISAAC J. SCHWATT.

Until the middle of the last century, mathematics had been developed only from the top, so to speak; but during the last decades, the efforts of some of the ablest mathematicians have been directed towards obtaining clearer conceptions of the foundations of mathematics.

Cantor, Weierstrass, Dini, Dedekind and many others have studied the fundamental conceptions of number and space, and their work has resulted in more accurate ideas about these conceptions.

With all the advance in the knowledge of mathematics, with our more thorough conceptions of the foundations of the science, and with the resulting tendencies to change the methods of presenting its various branches, the effect on the student's knowledge of the subject is far from encouraging.

During the last few years, quite a little unfavorable criticism has been made on the efficiency of our schools. Many of these criticisms refer to the results we now obtain as compared with those obtained before. We do not concern ourselves with the criticisms made by skeptics; they have been, and they always will be with us. Nor ought we to take seriously the statement by George Bernard Shaw: "Those who can, do; and those who cannot, teach." But there have been criticisms made by members of our own craft. Professor Charles W. Larned, of the United States Military Academy, in an article entitled, "The Inefficiency of our Public Schools," *North American Review*, September, 1908, tells us that out of 314 candidates who submitted

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